Introduction to Rheology in Sea Ice Models

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Outline

- Sea Ice as an important climate factor
- Rheology in Sea Ice
- Sea Ice models – a brief introduction
- Recent research efforts in Sea Ice rheology
- Summary

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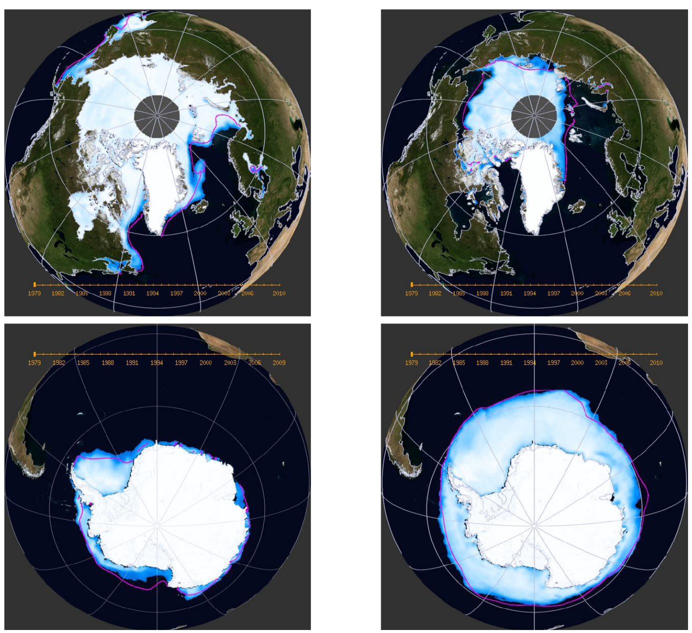
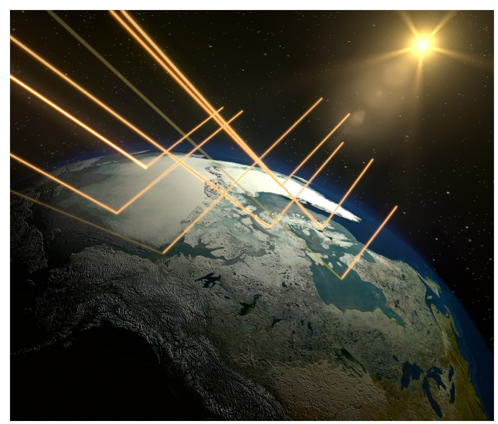
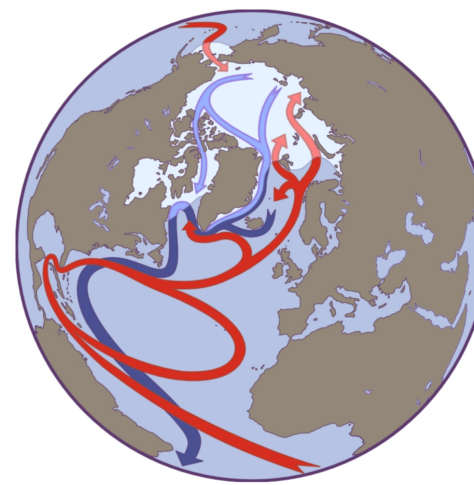
Sea Ice – Remote but Important

- Key component of the Earth's Climate System
  - Albedo feedback mechanism
  - Insulating effect of Atmosphere-Ocean interaction
  - Sustain of thermohaline circulation, generation of water masses

March September



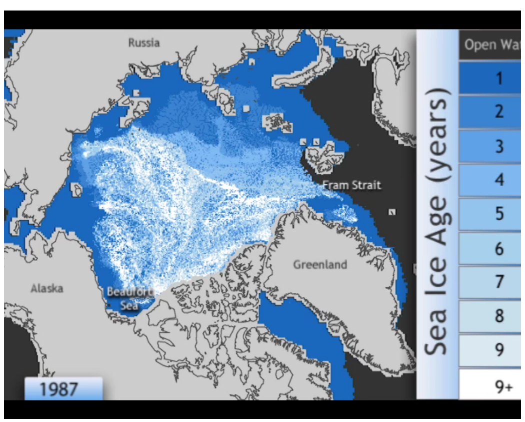


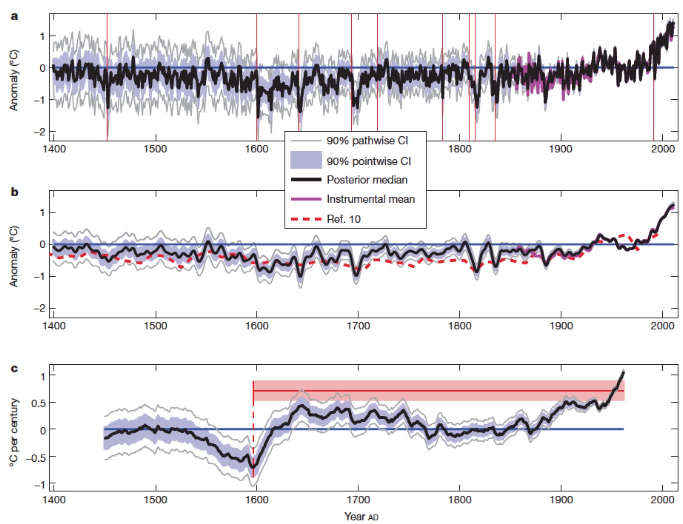


http://www.nsidc.org

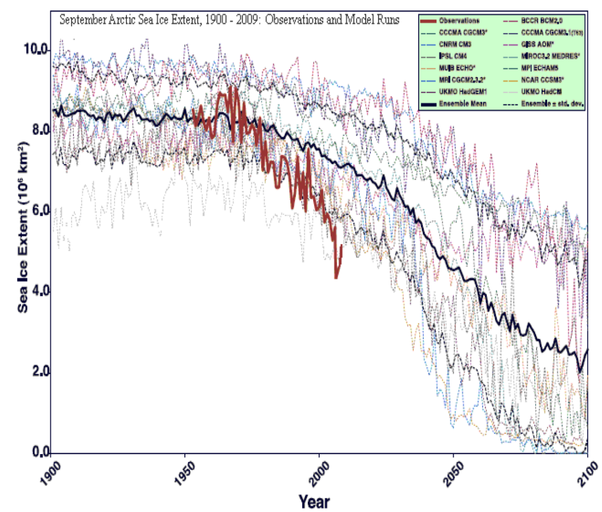
Sea Ice as a Hot Research Topic

- Sea Ice governs key polar processes with global impact
- Key Research Issues:
  - Rapid shrinkage of Arctic Sea Ice & growth of Antarctica Sea Ice
  - Unprecedented* warming of polar regions
  - Changing polar atmospheric circulation and ocean conditions
  - Future fate of Arctic & Antarctic Sea Ice





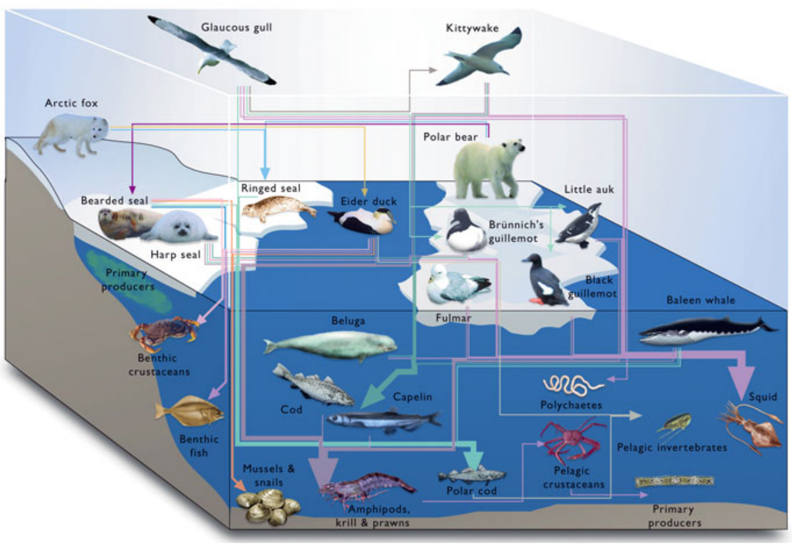
Tingley & Huybers, Nature, 2013

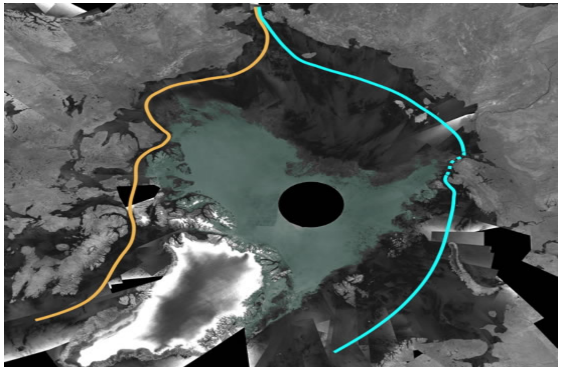


http://www.ipcc.ch

Sea Ice's Significance in Real Life

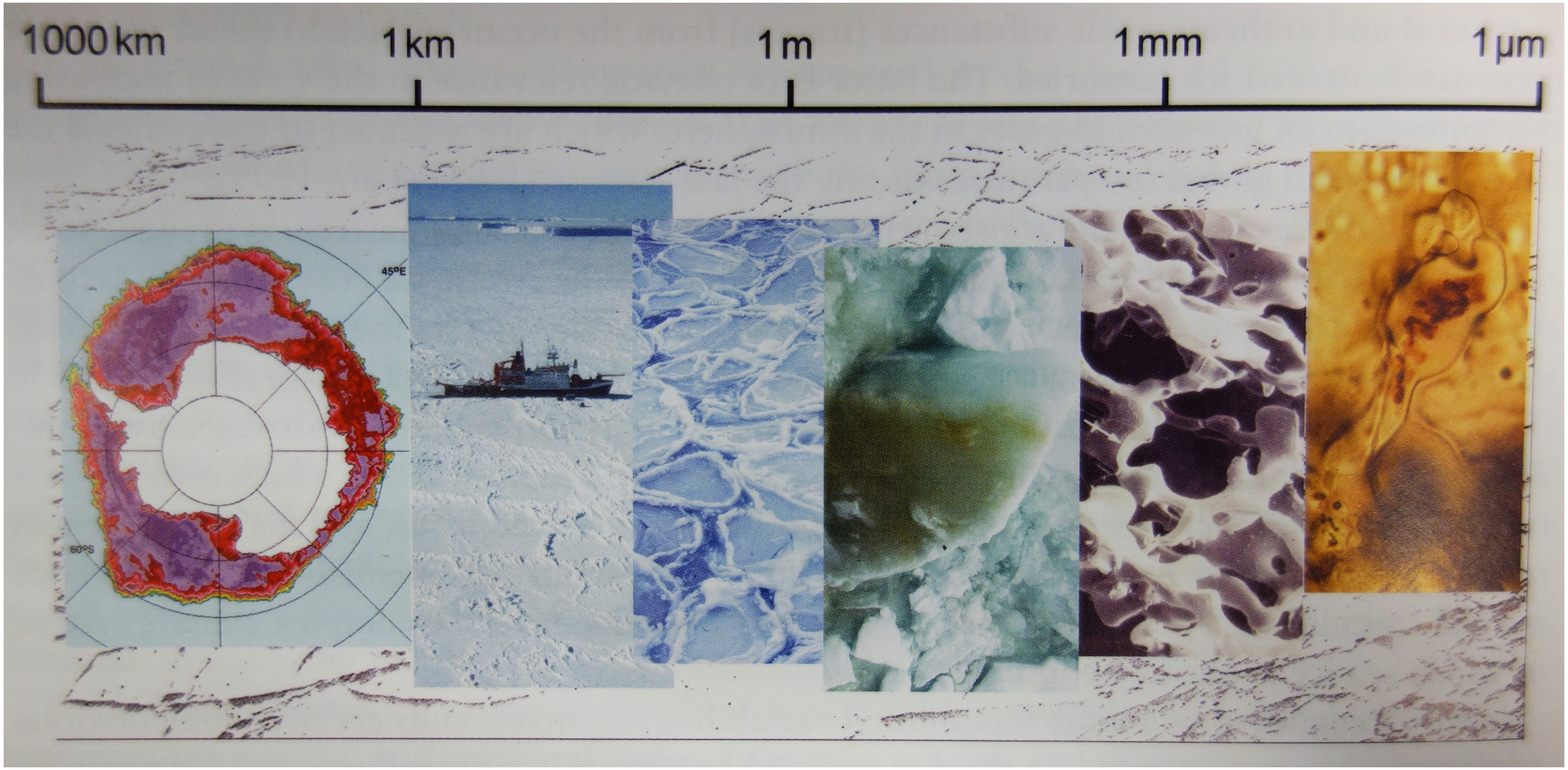
- Ecological impacts of Sea Ice changes
- Shipping routes through Arctic Ocean





Two German commercial vessels cross the Arctic's Northeast Passage

Multi-Scale of Sea Ice Processes

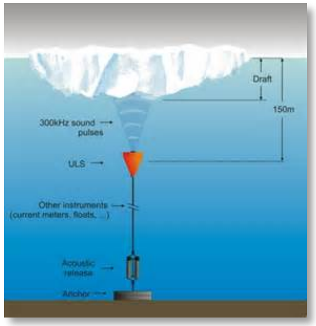


Thomas and Dieckman, Sea Ice, 2nd ed.

Observing / monitoring Sea Ice

- Satellite
  - Sea Ice emissivity modeling (SMMR/SMMI/AMSR)
  - Radar altimetry

- Buoys
  - Ice motion/growth/melt
  - ATM temperature/pressure/wind, SST, etc
- ULS (Upward Looking Sonar)
  - Sea Ice thickness, ocean conditions
- Field work, submarine, etc



Sea Ice Formation

Frazil ice

Rough ocean
(Pancake cycle)

Pancake ice
↓
Rafting or ridging
↓
Cementing and consolidation

Calm ocean
(Congelation growth)

Grease ice
↓
Nilas
↓
Rafting
↓
Congelation ice

Sheet ice

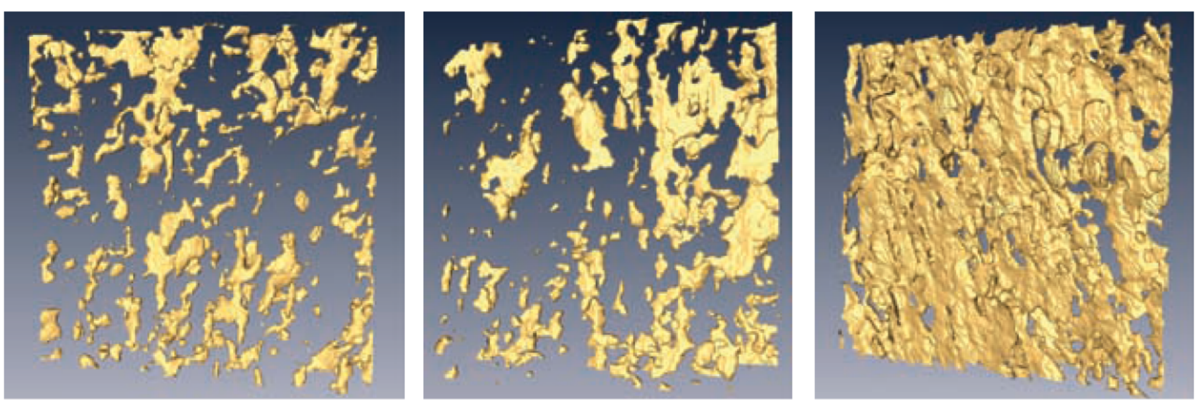
Smooth-bottomed: Grease ice pathway
Rough-bottomed: Pancake ice pathway

Micro-Structure of Sea Ice & Porosity

5 mm

Porosity affects:
- Compressibility, strength
- Premeability
- Ecological characteristics

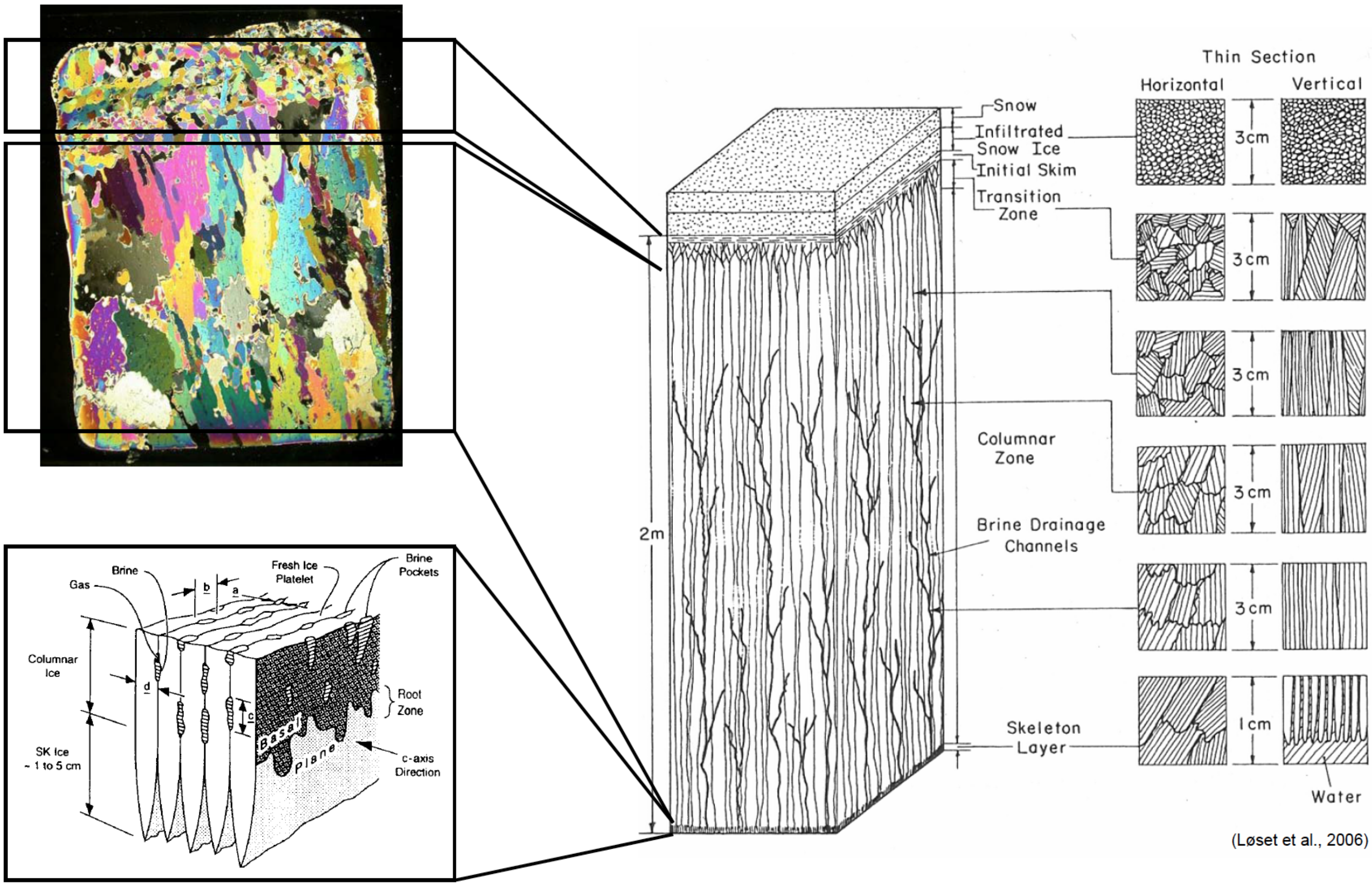
Porosity affected by:
- Temperature
- Relative position in Ice



(a) $T = -15\,^{\circ}\mathrm{C},\ \phi = 0.033$ $T = -6\,^{\circ}\mathrm{C},\ \phi = 0.075$ $T = -3\,^{\circ}\mathrm{C},\ \phi = 0.143$

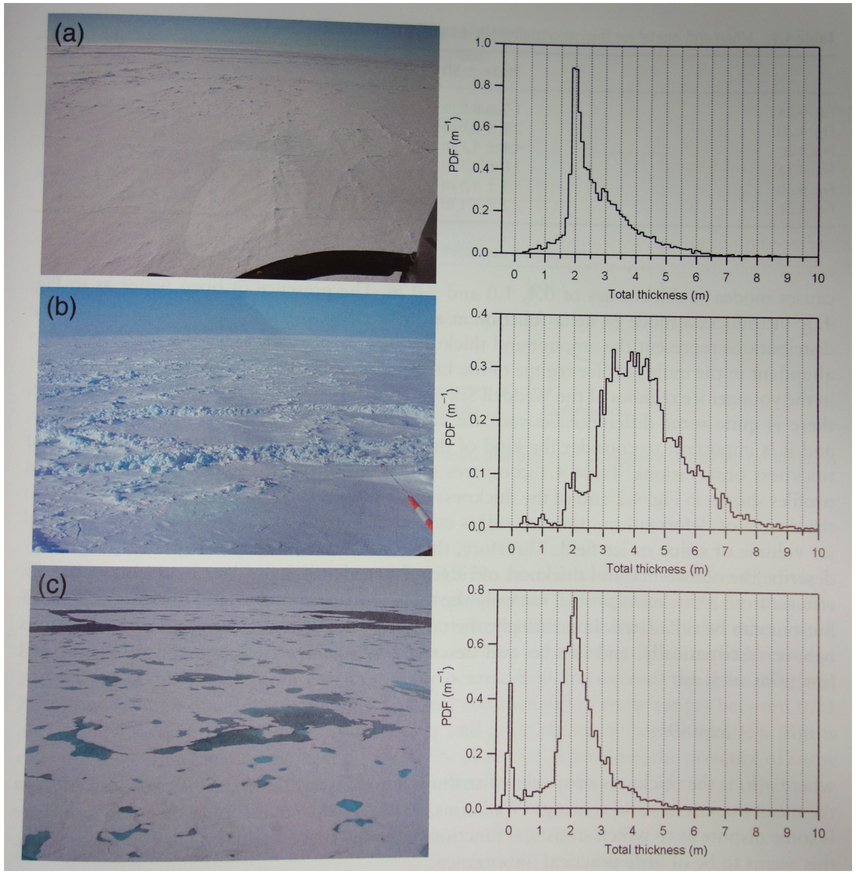
Golden et al., GRL, 2007

Vertical Structure of Sea Ice



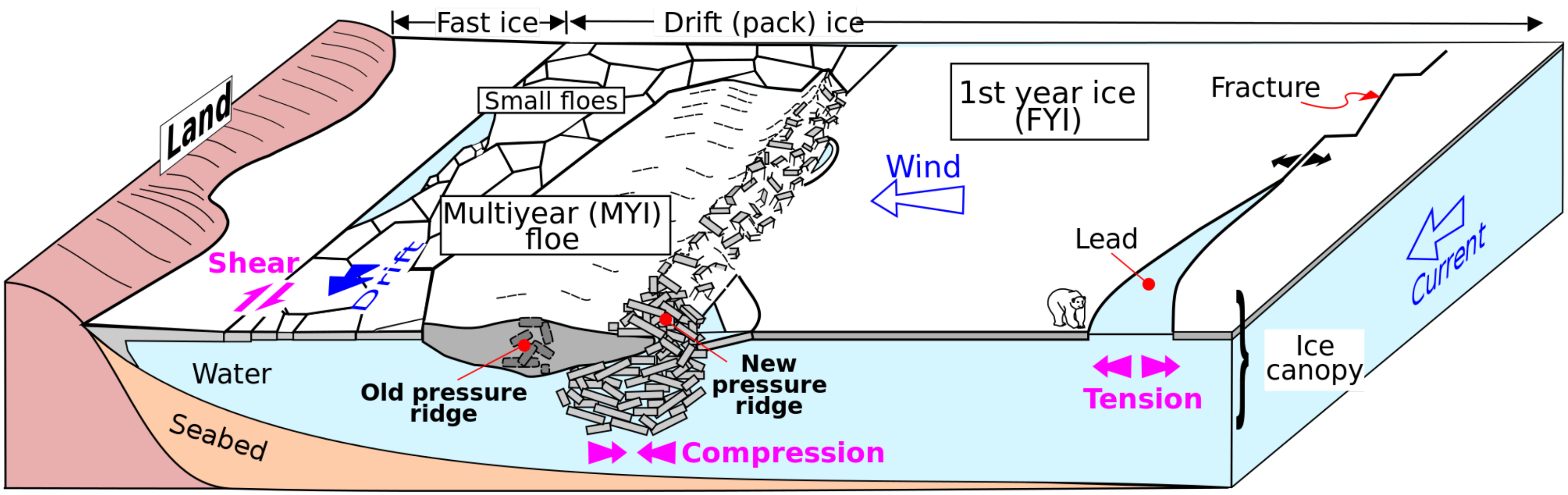
(Løset et al., 2006)

Sea Ice Thickness Distribution

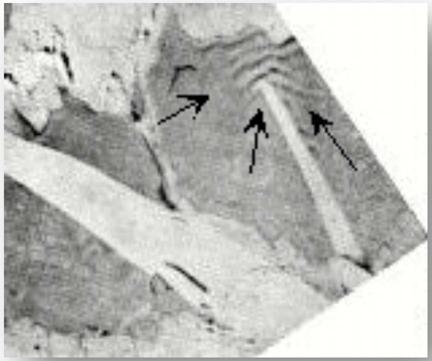


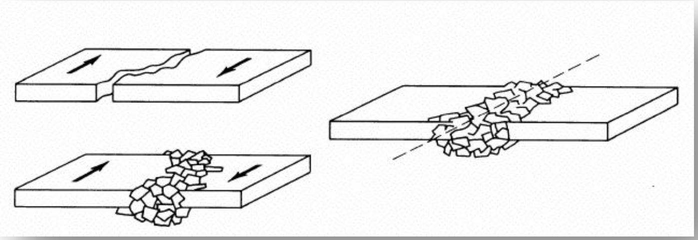
- (a) FY Sea Ice in Weddell Sea
- (b) MY Sea Ice in Lincoln Sea
- (c) Second-Year Sea Ice on North Pole (summer)

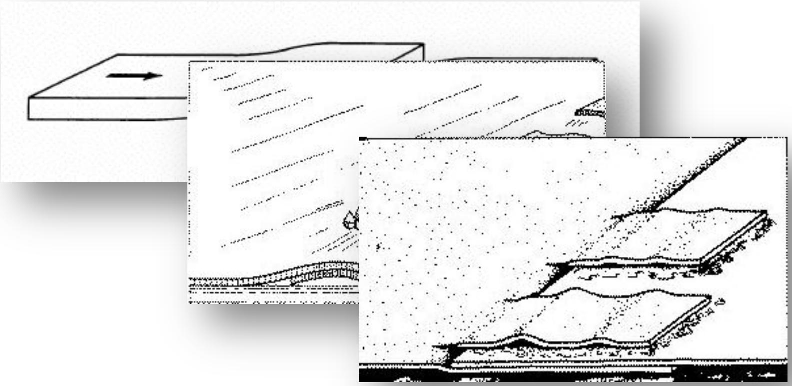
Sea Ice Dynamics



Sea Ice Deformation







Sea Ice Ridges & Leads

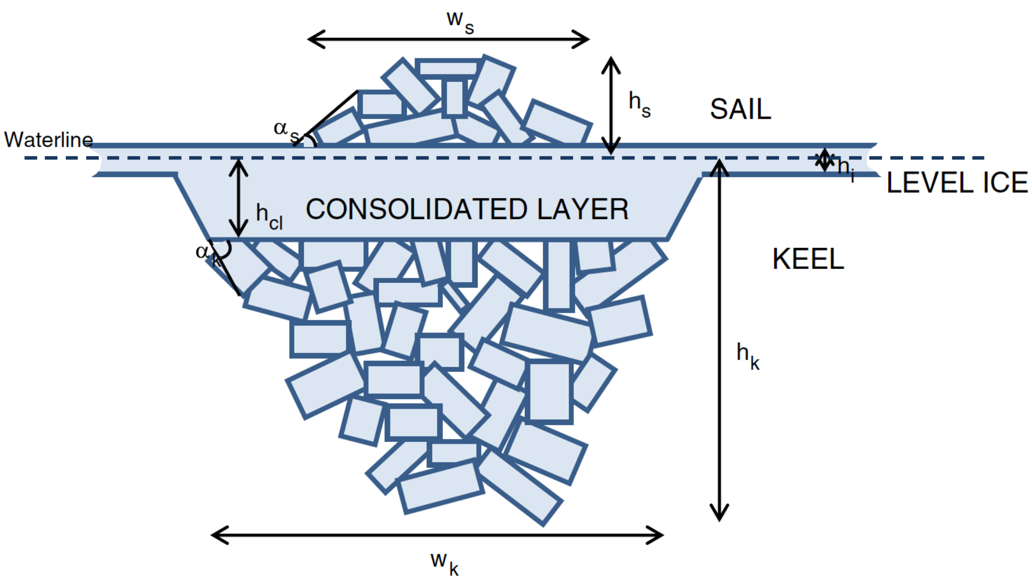
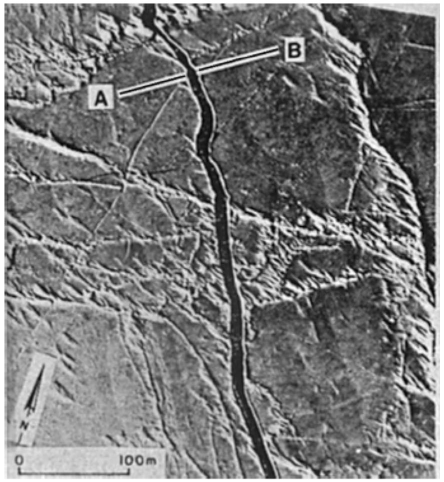
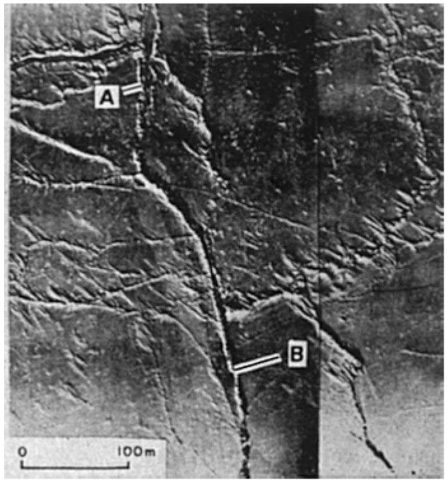


Table 6
Key statistical values for ratios of sail width to sail height, keel width to sail width, keel width to sail height, and keel width to keel depth.

	w_k/w_s	w_s/h_s	w_k/h_s	w_k/h_k
Mean	6.75	3.75	20.91	4.85
Max	35.89	9.62	86.67	16.67
n	165	130	152	149
Standard deviation	4.60	1.88	12.93	2.65
CoV	68%	50%	62%	55%

Strub-Klein and Sudom, CRST, 2012





Hibler, Engr. Frac. Mech., 2000

Importance of Sea Ice Deformation

- Generation of dense, MY Sea Ices
  - Key to the sustain of perennial ice cover in Arctic Ocean
- Generation of leads & polynya
  - Promotion of new Sea Ice growth & Atmosphere-Ocean interaction
  - Ecological significances
- Transformation of Sea Ice structure
  - Change of porosity, salinity, etc.
  - Interaction w/ Atmosphere and Ocean is affected
- Sink for kinematic energy
  - Transition to potential energy & internal energy (through friction)

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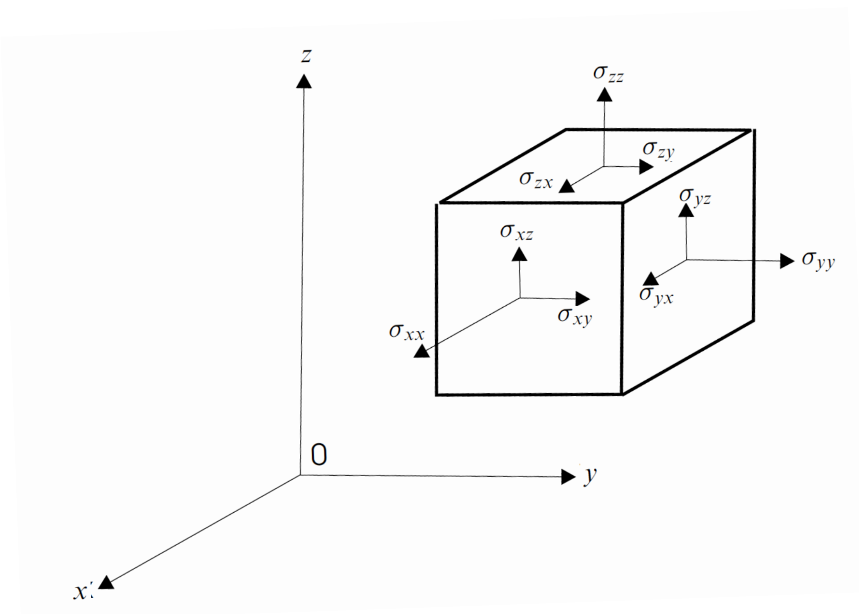
Rheology – Stress Tensor

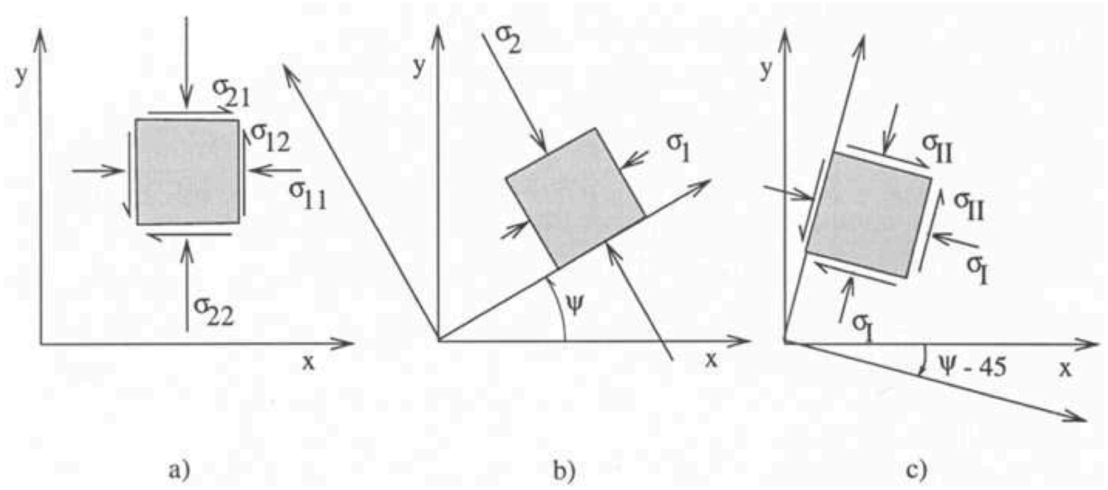


$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

Comments:

- Derived from continuum mechanics
- Normal stress & shear stress
- Symmetry of the tensor
- 2-D case for Sea Ice

Stress Tensor Transformations



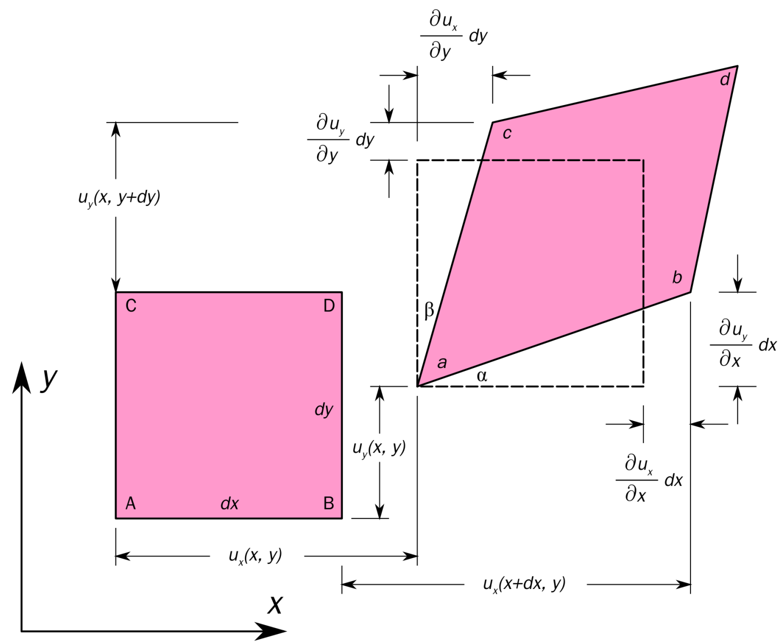
$$A = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \qquad B = \begin{bmatrix} \sigma_{1} & \\ & \sigma_{2} \end{bmatrix} \qquad B = \begin{bmatrix} \sigma_{I} & \sigma_{II} \\ \sigma_{II} & \sigma_{I} \end{bmatrix}$$

$$\sigma_1 = \sigma_{\mathrm{I}} - \sigma_{\mathrm{II}}$$
$$\sigma_2 = \sigma_{\mathrm{I}} + \sigma_{\mathrm{II}}.$$

$$\sigma_{\mathrm{I}} = \frac{1}{2}(\sigma_{11} + \sigma_{22})$$
$$\sigma_{\mathrm{II}} = \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2}.$$

- Basic: matrix congruence
  - $P^TAP=B$
  - Matrix trace is kept
- For 2-D Stress Tensor:
  a) Original coordinate
  b) Principle stress coordinate
  c) Stress invariant coordinate

Rheology – Strain Rate Tensor



$$\varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} \end{bmatrix}$$

$$\dot{\varepsilon}_{\mathrm{I}} = \dot{\varepsilon}_{11} + \dot{\varepsilon}_{22}$$

$$\dot{\varepsilon}_{\mathrm{II}} = \sqrt{(\dot{\varepsilon}_{11} - \dot{\varepsilon}_{22})^2 + 4\dot{\varepsilon}_{12}^2}.$$

Rheology – A Pragmatic View

<table>
<tr><td rowspan="4">Continuum mechanics</td><td rowspan="2">Solid mechanics</td><td colspan="2">Elasticity</td></tr>
<tr><td>Plasticity</td><td rowspan="2">Rheology</td></tr>
<tr><td rowspan="2">Fluid mechanics</td><td>Non-Newtonian Fluid</td></tr>
<tr><td colspan="2">Newtonian Fluid</td></tr>
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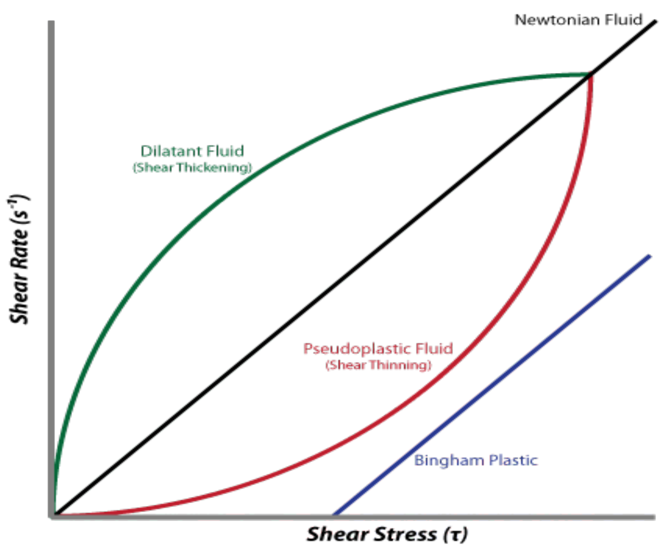
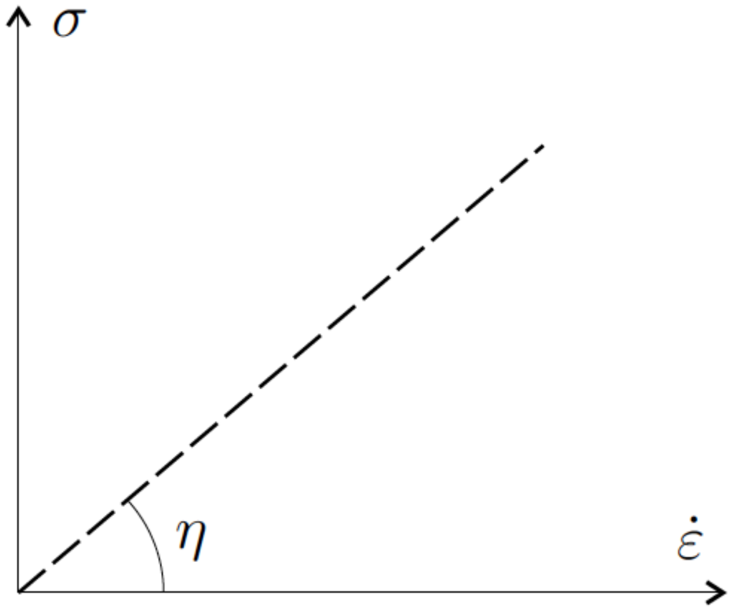


Examples of non-Newtonian fluids:

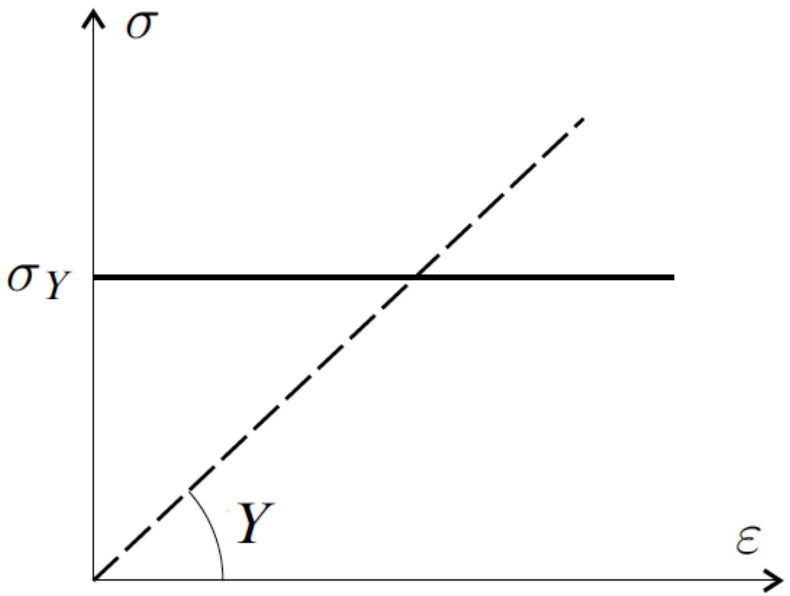
- Toothpaste
- Ketchup
- Sea Ice
- Mudslides

Basic Rheology – Viscous and Plastic

Viscous Fluid

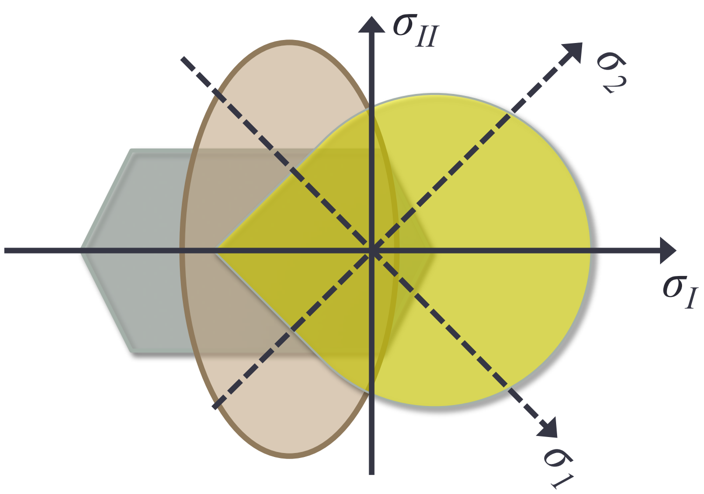


Plasticity



Rheology – Yield Curve for Plastics

- Yield Curve: a **convex** region in the stress tensor plane
  - Defines extreme values of stress when the plastic fails
  - Usually defined under stress-invariant coordinate or principal stress coordinate
  - When under stress-invariant coordinate:
    - Contains the origin
    - Symmetric over axis: σ_I
    - When containing positive side of σ_I:
      - Tensile strength is allowed
    - When containing negative side of σ_I:
      - Compressive strength is allowed
- Define function F on σ
  - Within yield curve, $F < 0$



General Sea Ice Rheology (1)

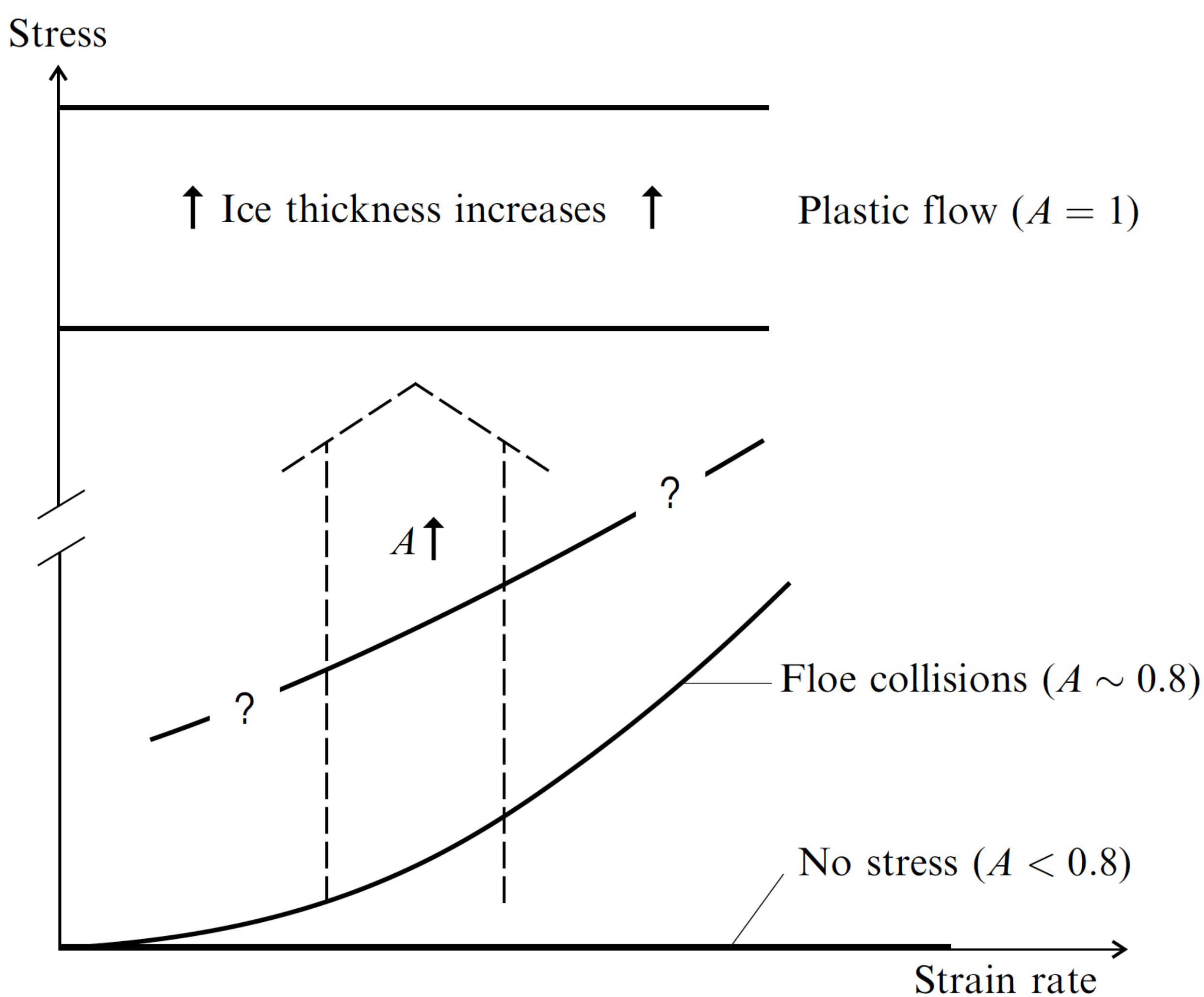
Characteristics

- Floe collision and breakup generates stress
- Shear friction exists between floes
- Friction generated in pressure ice formation
- Potential energy production in pressure ice formation

Implications in Rheology

- Weak floe interaction at low concentration levels
- Tensile strength is small
- Shear strength is significant
- Shear strength smaller than compressive strength
- Yield strength > 0

General Sea Ice Rheology (2)



Free Drift Scheme

- Ignore any internal forces of Sea Ice
  - No rheology in effect
- Problem:
  - No mechanism for characterizing internal stress for convergence
  - Too much Sea Ice will build up
- Corrections is applied. Set velocities to zero when:
  - Sea Ice thickness exceeds threshold
  - Thinner ice is advected towards thicker ice

Sea Ice Rheology – VP

- Viscous-Plastic (VP) proposed in [Hibler, JPO, 1979]
- Combine free-drift scheme and plastic scheme
  - Viscous behavior under small strain rate
  - Rigid, plastic behavior for large strain rate
  - Viscous law applies w/i the yield curve

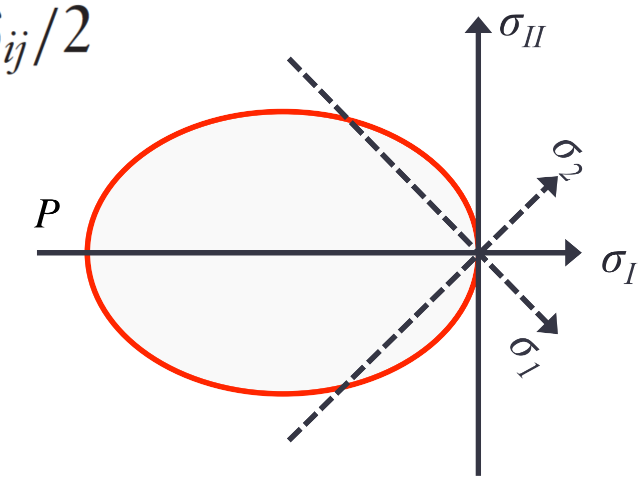
$$\sigma_{ij} = 2\eta\dot{\varepsilon}_{ij} + [\zeta - \eta]\dot{\varepsilon}_{kk}\delta_{ij} - P\delta_{ij}/2$$

- Choice for yield curve: ellipse

$$\zeta = P/2\Delta, \quad \eta = \zeta/e^2$$

$$P = P^* h \exp[-C(1 - A)]$$

$$\Delta = \sqrt{\dot{\varepsilon}_{\mathrm{I}}^2 + (\dot{\varepsilon}_{\mathrm{II}}/e)^2}$$



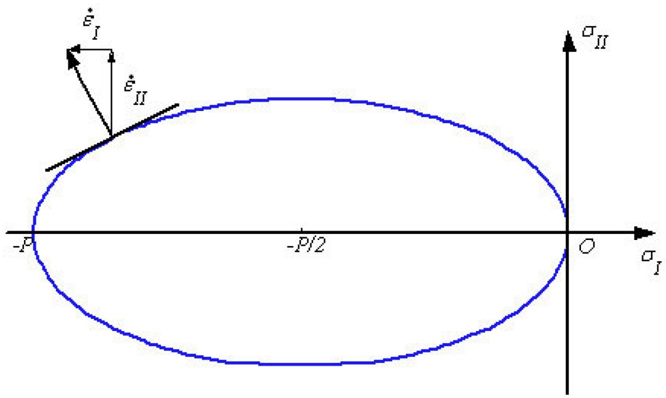
$$F(\sigma_1, \sigma_2) = \left(\frac{\sigma_1 + \sigma_2 + P/2}{P/2}\right)^2 + \left(\frac{\sigma_1 - \sigma_2}{P/2e}\right)^2$$

Sea Ice Rheology – VP (2)

- Normal Flow Rule:
  - Strain-rate vector is normal to the yield curve at the failure point

$$\dot{\varepsilon}_1 = \lambda \frac{\partial F}{\partial \sigma_1} = \lambda \left(\frac{2(\sigma_1 + \sigma_2 + P/2)}{(P/2)^2} + \frac{2(\sigma_1 - \sigma_2)}{(P/2e)^2} \right)$$

$$\dot{\varepsilon}_2 = \lambda \frac{\partial F}{\partial \sigma_2} = \lambda \left(\frac{2(\sigma_1 + \sigma_2 + P/2)}{(P/2)^2} - \frac{2(\sigma_1 - \sigma_2)}{(P/2e)^2} \right)$$



- Practical usage:

$$\zeta = \min\left(\frac{P_{\max}}{2\max(\Delta, \Delta_{\min})}, \zeta_{\max} \right)$$

Table 4.2. Parameters of the viscous–plastic ice rheology of Hibler (1979).

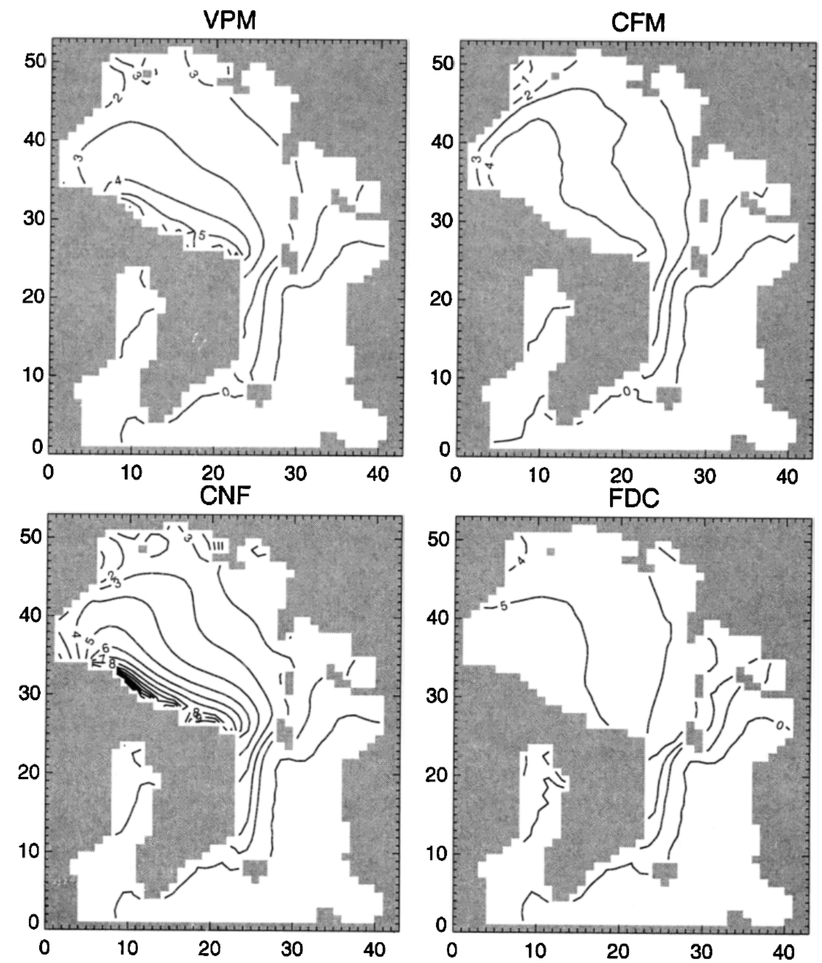
Parameter	Notation and standard	Range
Compressive strength	$P^* = 25\,\mathrm{kPa}$	10–100 kPa
Yield ellipse aspect ratio	$e = 2$	$1 < e \ll \infty$
Compaction hardening	$C = 20$	$C \gg 1$
Maximum creep	$\Delta_0 = 2 \times 10^{-9}\,\mathrm{s}^{-1}$	$\Delta_0 < 10^{-7}\,\mathrm{s}^{-1}$

Cavitating Fluid

- Proposed by (Flato & Hibler, 1990)
- Construction from VP rheology
  - Compress the ellipse on the direction of σ_{II}, to form a line
  - Do not bear any shearing
- Advantages:
  - Simple formation of momentum equations
- Disadvantages:
  - Failure to characterize shear-induced deformation

Rheology Algorithms – A Comparison

- SIMIP (1997~98)
  - 1979~1995 yrs are simulated



Kryescher, et al., JGR, 2000

- Viscous-Plastic (VPM)
- Cavitating Fluid (CFM)
- Compressible Newtonian Fluid (CNF)
- Free-Drift (FDC)

Other Plastic Rheology Schemes

- Many plastic rheology schemes exists
  - Some allow biaxial tensile strength
  - No general conclusion for which is the best

Table 3. Comparisons of Model and Buoy Velocities Based on Daily, 5-day, and 10-day Averages[a]

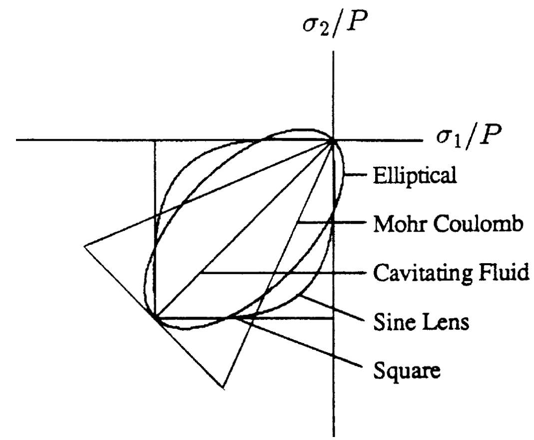
Case	Mean Model Ice Speed, cm s^{-1}	Model Bias[b]	Error SD[c]	Correlation, %
Ellipse	7.8 (6.9) [6.1]	−0.1 (0.6) [0.8]	7.1 (5.0) [4.0]	73 (80) [83]
MCE	7.4 (6.5) [5.8]	−0.5 (0.2) [0.5]	7.0 (4.9) [3.9]	73 (80) [83]
Teardrop	7.1 (6.2) [5.4]	−0.8 (−0.1) [0.1]	7.0 (4.9) [3.8]	72 (79) [82]
Lens	7.9 (6.9) [6.0]	−0.0 (0.6) [0.7]	7.0 (5.0) [3.9]	74 (81) [83]
Teardrop 1	7.4 (6.5) [5.7]	−0.5 (0.2) [0.4]	7.0 (4.9) [3.8]	73 (80) [83]
Lens 1	8.2 (7.2) [6.2]	0.3 (0.9) [0.9]	7.0 (5.0) [4.0]	75 (81) [84]

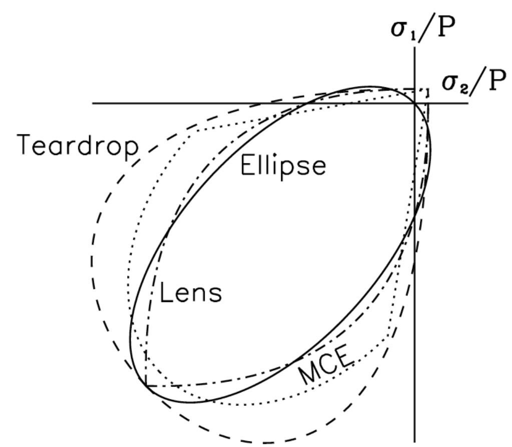
[a]The 5-day averages are given in parentheses, and the 10-day averages are given in brackets.
[b]During 1993–1997, there are 38,466 daily mean buoy velocities with a mean speed of 7.9 cm s^{-1}, 7348 5-day mean buoy velocities with a mean speed of 6.3 cm s^{-1}, and 3616 10-day mean buoy velocities with a mean speed of 5.3 cm s^{-1}.
[c]SD is standard deviation.

Table 4. Modeled and Observed Ice Drafts Compared Along Tracks of Four Submarine Cruises in 1993–1997[a]

Case	Mean Model Draft, m	Bias	Error SD	Correlation
Ellipse	2.01	0.04	0.71	0.44
MCE	2.13	0.16	0.75	0.47
Teardrop	2.01	0.04	0.69	0.51
Lens	1.91	−0.06	0.73	0.45
Teardrop 1	1.90	−0.07	0.69	0.50
Lens 1	1.83	−0.14	0.74	0.42

[a]There are 639 observed drafts with a mean of 1.97 m.





Zhang and Rothcock, JGR, 2005

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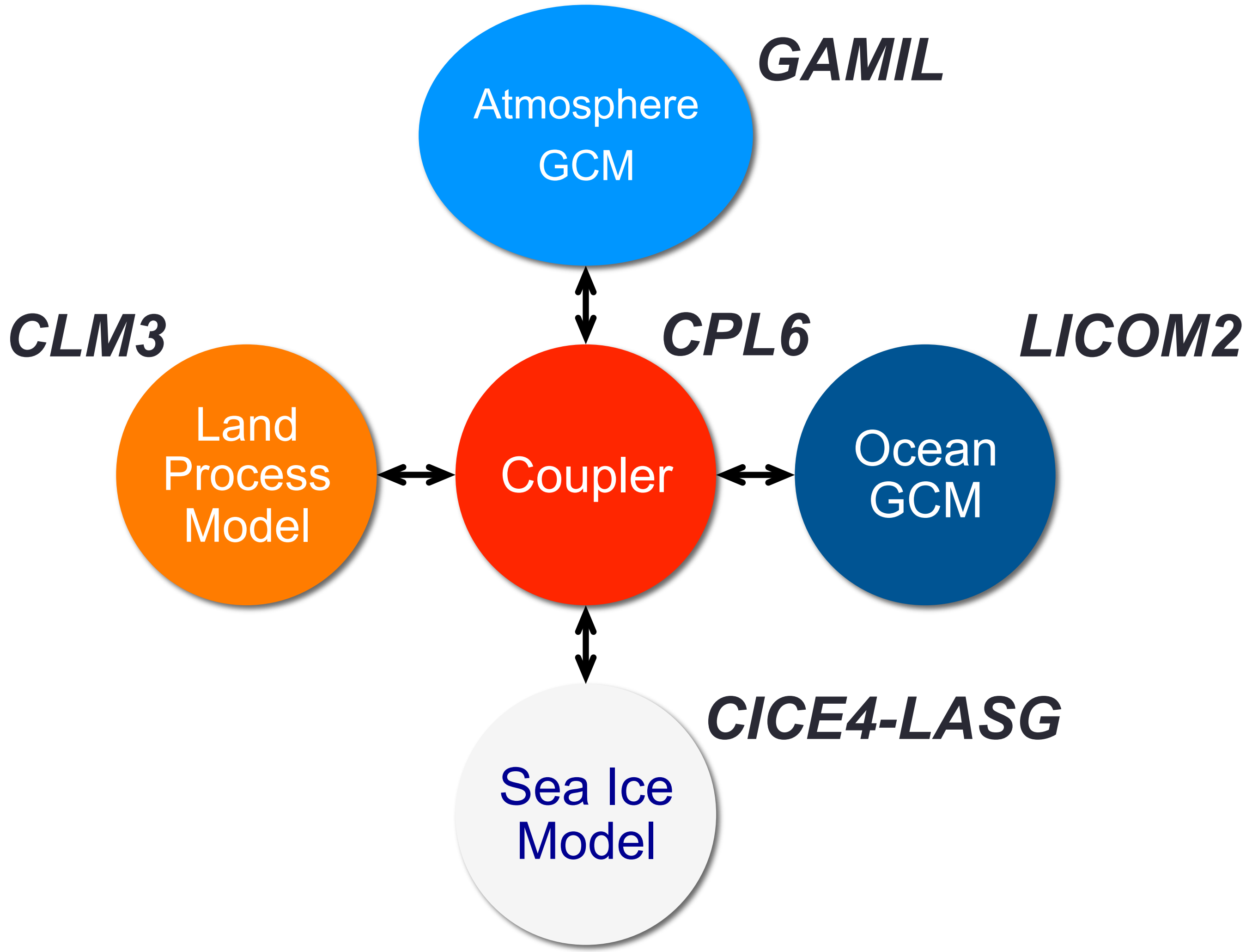
Sea Ice in Coupled Models



Glossary of Processes in Sea Ice Models

- Thermodynamics
  - Growth & melt of Sea Ice
  - Albedo & disposition of SW/LW
  - Thermal conductivity within Sea Ice
- Dynamics
  - Thickness distribution changes due to rafting/ridging
  - Sea Ice movement governed by external/internal forces
- Coupling w/ Atmosphere and Ocean
  - Momentum exchange through boundary layer processes
  - Flux exchanges:
    - Latent/sensible heat
    - Water: precipitation, evaporation, sublimation
    - Salinity (through brine expulsion)

Governing Equations in Sea Ice Models

- Thickness distribution equation

$$\frac{\partial g}{\partial t} = -\nabla \cdot (g\mathbf{u}) - \frac{\partial}{\partial h}(fg) + \boxed{\psi}$$

Mechanical Redistribution

Advection

Thermodynamic growth/melt

- Momentum equation

$$m\frac{\partial \mathbf{u}}{\partial t} = \boxed{\nabla \cdot \sigma} + \boxed{\vec{\tau}_a} + \boxed{\vec{\tau}_w} - \hat{k} \times mf\mathbf{u} - mg\nabla H_\circ$$

Air Drag

Coriolis Term

Internal Stress

Ocean Surface Tilt

Ocean Drag

Contribution of Mechanical Redistribution

$$\frac{\partial g}{\partial t} = -\nabla \cdot (g\mathbf{u}) - \frac{\partial}{\partial h}(fg) + \boxed{\psi}$$

Mechanical Redistribution

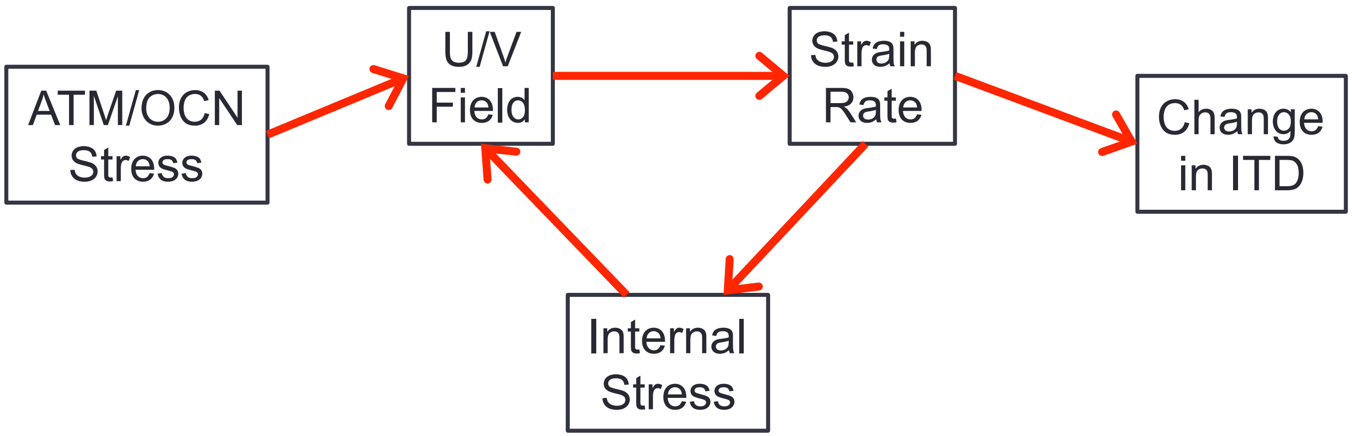
- For convergence:
  - Fill spaces with no Sea Ice
  - Generation of thick Sea Ice ridges
  - Shift /create distribution in thicker categories

- For divergence
  - Generation of leads (i.e., open water)
  - Create/enlarge thickness distribution $g(h)$ at $h=0$

Contribution of Internal Stress

$$m\frac{\partial \mathbf{u}}{\partial t} = \boxed{\nabla \cdot \sigma} + \boxed{\vec{\tau}_a} + \boxed{\vec{\tau}_w} - \hat{k} \times mf\mathbf{u} - mg\nabla H_\circ$$

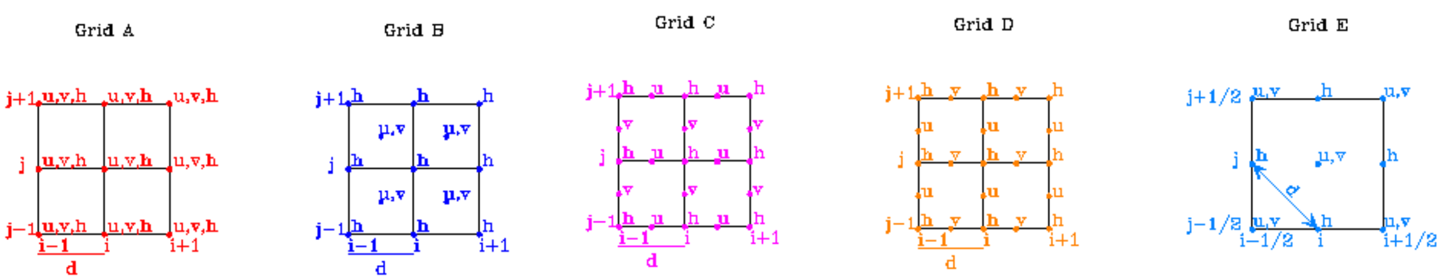
Internal Stress



Spatial Discretization of Sea Ice

- General Orthogonal Coordinates
  - Shifted Pole
  - Displaced Pole
  - Tri-pole

- Grid Staggering:
  - Arakawa B grid
  - Arakawa C grid



Solution of VP-based Momentum Eq.

- Explicit method:
  - Impractical due to limited time step size: ~10s for 100km
- Implicit method
  - Non-linear momentum equations to be solved

$$\rho h^{n-1}\frac{\partial u^n}{\partial t} = \rho h^{n-1} f v^n - \tau_{wu}^n + \frac{\partial \sigma_{11}{}^n}{\partial x} + \frac{\partial \sigma_{12}{}^n}{\partial y} + r_{*u}^n,$$

$$\rho h^{n-1}\frac{\partial v^n}{\partial t} = -\rho h^{n-1} f u^n - \tau_{wv}^n + \frac{\partial \sigma_{22}{}^n}{\partial y} + \frac{\partial \sigma_{12}{}^n}{\partial x} + r_{*v}^n,$$

$$\sigma_{11}^n = \zeta^n\left(\frac{\partial u^n}{\partial x} + \frac{\partial v^n}{\partial y}\right) + \eta^n\left(\frac{\partial u^n}{\partial x} - \frac{\partial v^n}{\partial y}\right) - \frac{P}{2},$$

$$\sigma_{22}^n = \zeta^n\left(\frac{\partial v^n}{\partial y} + \frac{\partial u^n}{\partial x}\right) + \eta^n\left(\frac{\partial v^n}{\partial y} - \frac{\partial u^n}{\partial x}\right) - \frac{P}{2},$$

$$\sigma_{12}^n = \eta^n\left(\frac{\partial u^n}{\partial y} + \frac{\partial v^n}{\partial x}\right).$$

Linearization & solution of large linear system is inevitable

*Optional implicit formulation on Arakawa C grid

Rheology Scheme – EVP

- Elastic-Viscous-Plastic model (EVP) [Hunke and Dukowicz, JPO, 1997]
  - Introduce artificial elastic term in stress tensor

$$\boxed{\frac{1}{E}\frac{\partial \sigma_{ij}}{\partial t}} + \frac{1}{2\eta}\sigma_{ij} + \frac{\eta-\zeta}{4\zeta\eta}\sigma_{kk}\delta_{ij} + \frac{P}{4\zeta}\delta_{ij} = \dot{\epsilon}_{ij}$$

  - When elastic wave damps, real solution is attained
  - Use sub-step iterations to allow explicit integration

$$\frac{(\sigma_1^s - \sigma_1^{s-1})}{\Delta t_e} + \frac{\sigma_1^s}{2T} = \frac{\zeta^{s-1}(\dot{\epsilon}_{11}^{s-1} + \dot{\epsilon}_{22}^{s-1})}{T} - \frac{P}{2T},$$

$$\frac{(\sigma_2^s - \sigma_2^{s-1})}{\Delta t_e} + \frac{e^2\sigma_2^s}{2T} = \frac{\zeta^{s-1}(\dot{\epsilon}_{11}^{s-1} - \dot{\epsilon}_{22}^{s-1})}{T},$$

$$\frac{(\sigma_{12}^s - \sigma_{12}^{s-1})}{\Delta t_e} + \frac{e^2\sigma_{12}^s}{2T} = \frac{\zeta^{s-1}\dot{\epsilon}_{12}^{s-1}}{T},$$

$$T = \zeta/E$$

$$N_{sub} * \Delta t_e = \Delta t$$

$$\Delta t_e < \frac{4e\Delta x}{(1+e^2)}\left(\frac{\rho h T}{\zeta}\right)^{\frac{1}{2}}.$$

EVP Scheme (2)

- Advantages:
  - Simple formulation with explicit integration scheme
  - Parallel machine friendly

- Widely used in Sea Ice models in climate research:
  - CICE4 hence CICE4-LASG
  - LIM3, CSIM5, etc

Mechanical Redistribution for ITD

- Use of strain rate to fill/generate open area
- Participation function: $a(h)=b(h)g(h)$
  - What part of the Sea Ice is involved in ridging/rafting, given $a(h)$
- Resulting distribution function: $n(h)$
  - What are the resulting thickness distribution after ridging/rafting, given $g(h)$ and $a(h)$
- Conservation of volume & energy

$$b(h) = \begin{cases} \frac{2}{G*}\left(1 - \frac{G(h)}{G*}\right) & \text{if } G(h) < G^* \\ 0 & \text{otherwise} \end{cases}$$

$$H_{\min} = 2h_n$$

$$H_{\max} = 2\sqrt{H^* h_n}$$

$H^*=50m$ for FY ice
$H^*=25m$ for MY ice

Triangular profile for keel shape

ITD of Ridge follows exponential distribution

No porosity considered

Outline

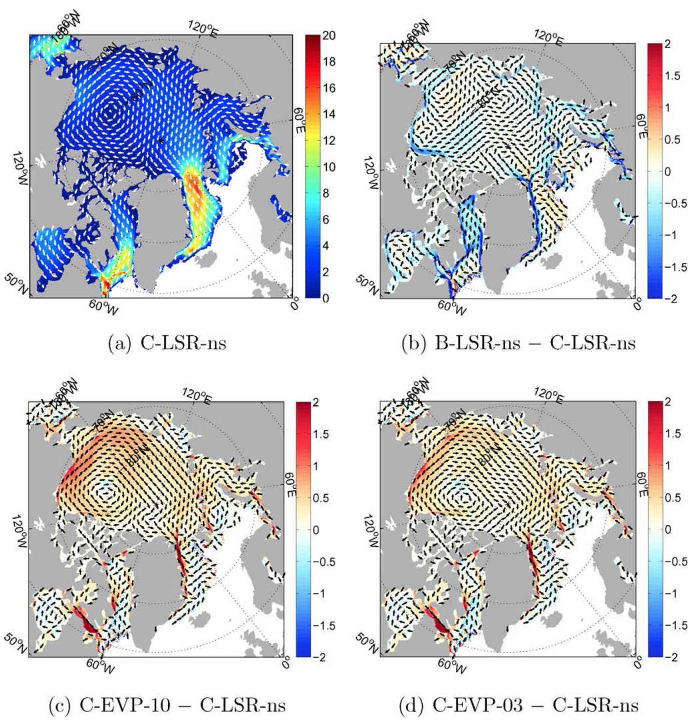
- Sea Ice as an important climate factor
- Rheology in Sea Ice
- Sea Ice models – a brief introduction
- **Recent research efforts in Sea Ice rheology**
- Summary

Recent Trends in Sea Ice Rheology

- Using classical VP scheme instead of EVP
- Anisotropy rheology
- Brittle rheology (instead of plastic rheology)
- Modeling of landfast Sea Ice

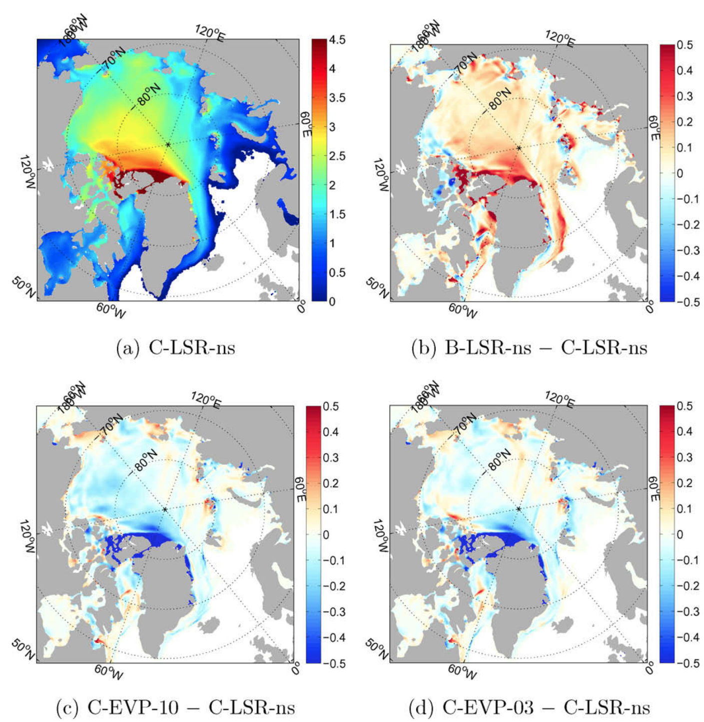
Choosing VP over EVP (1)

- Comparison of EVP and VP show large differences
  - Result's sensitivity to step size
  - Attributed to reduced viscosity due to EVP treatment for A ~ 1



(a) C-LSR-ns (b) B-LSR-ns − C-LSR-ns
(c) C-EVP-10 − C-LSR-ns (d) C-EVP-03 − C-LSR-ns

Velocity



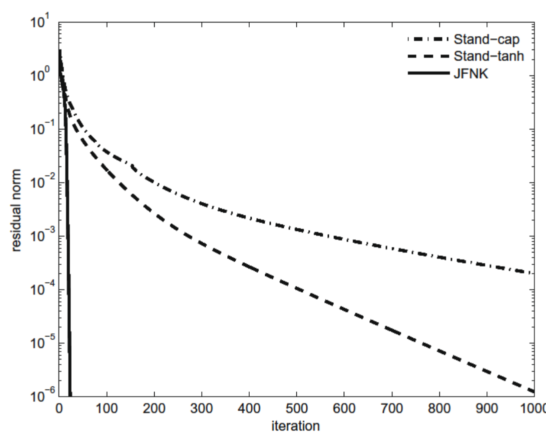
(a) C-LSR-ns (b) B-LSR-ns − C-LSR-ns
(c) C-EVP-10 − C-LSR-ns (d) C-EVP-03 − C-LSR-ns

Thickness

Losch et al., Ocean Modeling, 2010

Choosing VP over EVP (2)

- Design better implicit solvers to VP
  - Nonlinear solvers can solve VP with consistent convergence
  - How to solve linear systems leaves much potential for improvements
- General solver framework
  - SOR-based [Zhang and Hibler, JGR, 1997]
  - Picard solver and JFNK solver w/ GMRES



Lemieux et al., JCP, 2010

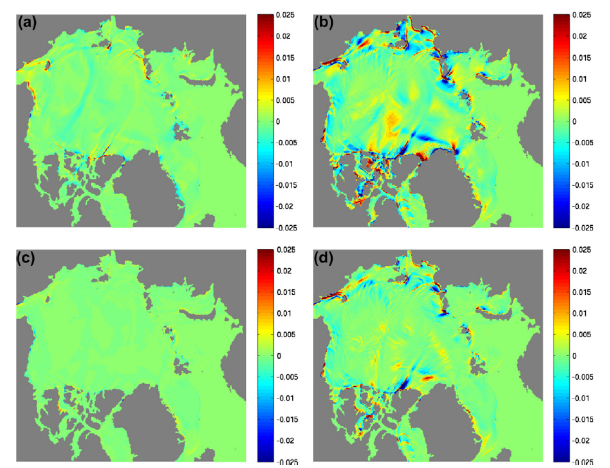


Fig. 5. Difference between the thickness field obtained with JFNK with $\gamma_{nl} = 0.5$ (a) or $\gamma_{nl} = 10^{-3}$ (c) and the reference solution. Difference between the thickness field obtained with the EVP with $N_{sub} = 120$ (b) or $N_{sub} = 1920$ (d) and the reference solution. The advective time step for the JFNK and EVP solvers is 20 min. To see the details, the thickness differences are capped to ± 2.5 cm.

Lemieux et al., JCP, 2012

Anisotropy in Sea Ice Modeling

- Basis: anisotropy is a dominant factor at finer scales (<10km)
  - Anisotrophy in ITD is prominent
  - Misaligned σ and $\dot{\varepsilon}$
  - Divergence creates leads
  - Convergence/shearing creates ridges
- Key to Sea Ice forecast
- Extended problem:
  - Anisotrophy affects boundary layer

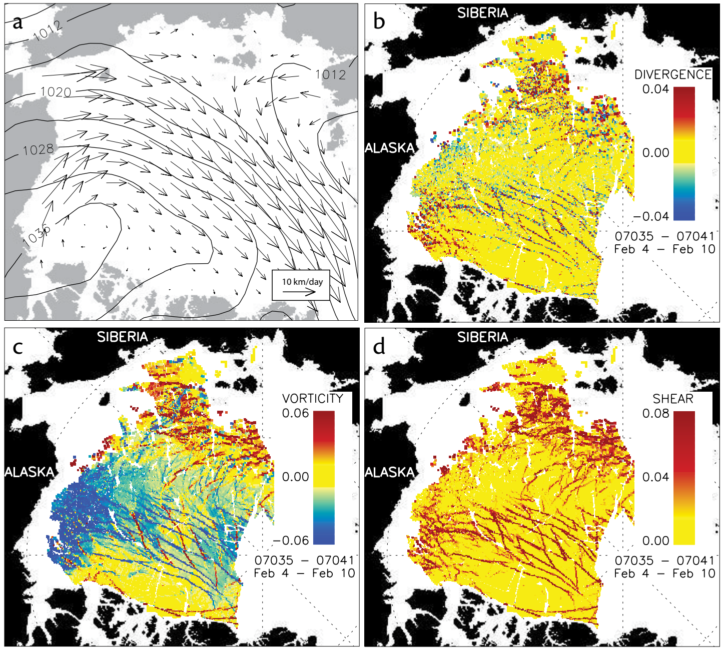
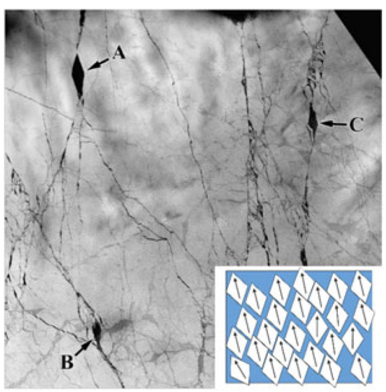


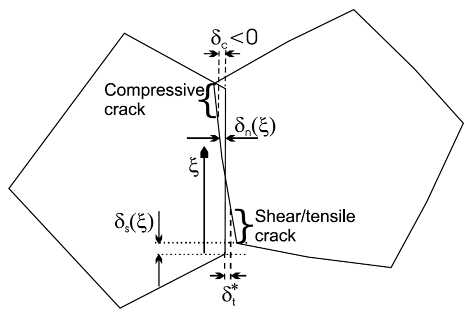
Figure 1. Large-scale mean ice motion and deformation of the Arctic Ocean ice cover between February 4 and February 10, 2007. The high-resolution ice deformation fields are derived from synthetic aperture radar imagery. (a) Mean vector field with superimposed sea level pressure contours (Interval: 4 hPa). (b) Divergence. (c) Vorticity. (d) Shear. Deformation computed at grid cell ~ 10 km on a side. Units: (/day).

Kwok and Sulsky, Oceanography, 2010

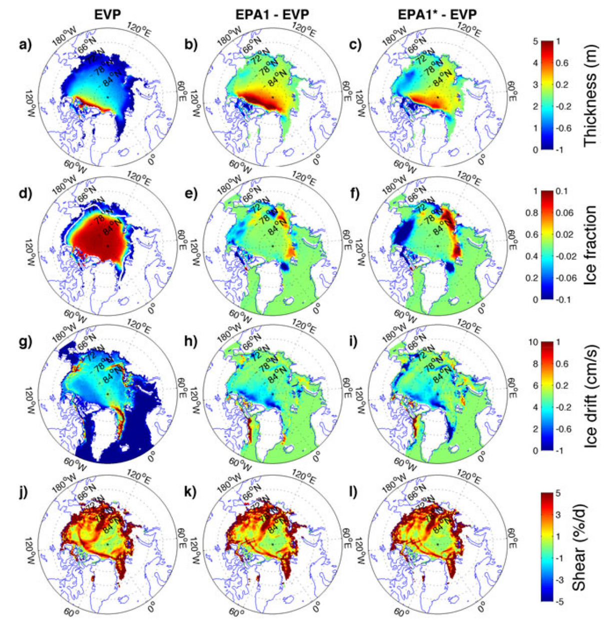
Anisotropy in Sea Ice Modeling (2)

- Description of anisotropy
  - Diamond-shaped floes [Wilchinsky and Feltham, J Non-Newtonian Fluid Mech., 2006]
  - Voronoi graph based floes
  - Statistical description of lead alignment





Wilchinksky et al., JGR, 2010



Tsamados et al., JGR, 2012

- Problems ahead:
  - Effective model integration through parameterization/simplification
  - Coherency with: anisotropic at high resolution & isotropic at low resolution

Elastic Brittle (EB) Rheology

- Motivation: Deformation cascades with long-range effects
- Modeling details:
  - Finite-Element Method
  - Quasi-static condition

$$\nabla . \underline{\underline{\sigma}} + \underline{\tau_a} + \underline{\tau_w} = 0$$

  - Elasticity

$$\sigma_{ij} = K \underline{\underline{D}} \epsilon_{ij}, \quad \text{with } \sigma_{ij} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix},$$

$$\underline{\underline{D}} = \frac{1}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad \text{and } \epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix}$$

  - Brittleness

$$K_0 = Yh \exp^{-\upsilon(1-c)}$$

$$K_i(n+1) = K_i(n) d_0$$

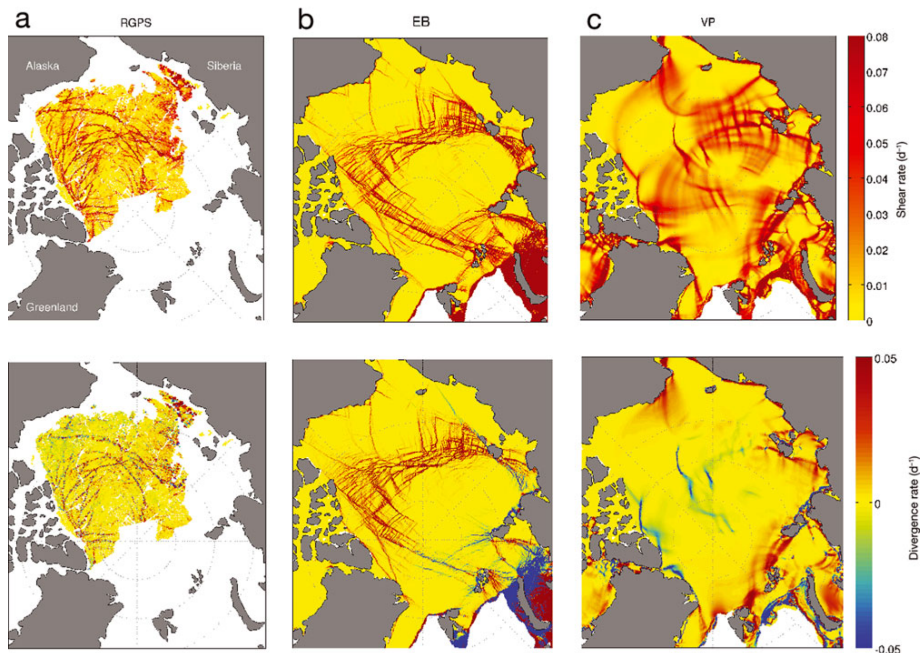


Fig. 3. Shear and divergence rate from (a) RGPS observations, (b) EB simulation and (c) VP simulation. The RGPS observations represented were obtained between 27 March and 1 April 2007. Strain rates from EB and VP simulations were computed between 27 and 30 March 2007, for a temporal scale of 3 days.

Girard et al., Annals Glaciology, 2011

Modeling Landfast Sea Ice

- Main methodology:
  - Add tensile/cohesion support to yield curve [Beatty and Holland, JPO, 2010]
  - Similar to other rheology supporting biaxial tensile forces (teardrop, etc.)

$$F(\sigma_1, \sigma_2) = \left(\frac{\sigma_1 + \sigma_2 + P - T}{P + T}\right)^2 + \left(\frac{\sigma_1 - \sigma_2}{P + T} e\right)^2 - 1 = 0$$

- Not applied to Sea Ice models yet

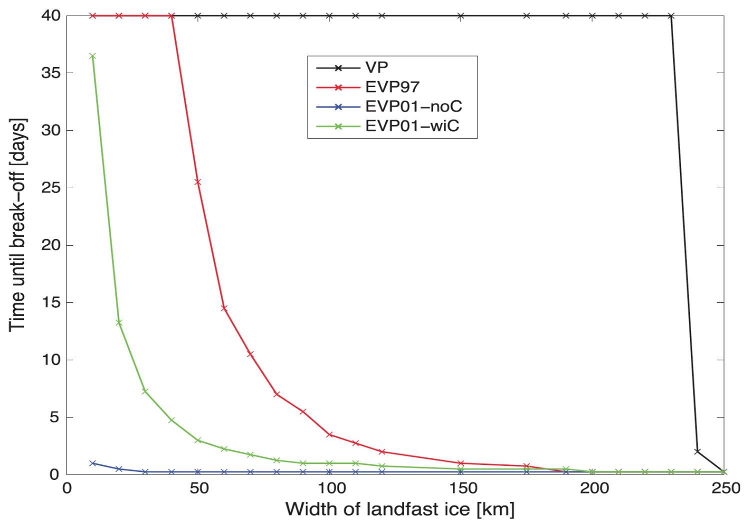
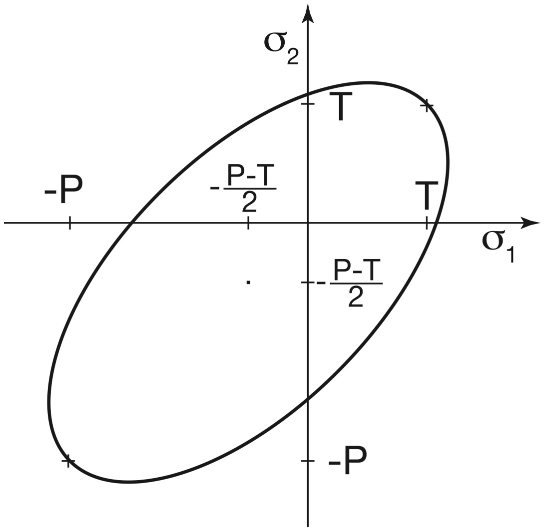


FIG. 9. Number of days until landfast ice of a certain width breaks off under the influence of offshore wind.



Outline

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Summary

- Sea Ice is an integral component of climate systems
  - Albedo effect, insulation effect, etc.
- Rheology is an important factor in Sea Ice
  - Unique characteristics than atmosphere or ocean
  - Governs key Sea Ice processes: ridging, interaction w/ land, etc
- Rheology is a hot topic in Sea Ice model development
  - Much uncertainty due to heavily parameterized/idealized rheology schemes
  - High resolution points to new directions for rheology
    - High-resolution needed for Sea Ice forecast
    - Anisotropy, finite-element (irregular element based methods), etc.
  - **BOTH scientific understanding & clever computational treatments are important !!!**

THANKS ! ANY QUESTIONS?